



NTNU

Norwegian University of
Science and Technology

Compiler Construction

Lecture 20: Reaching definitions

Michael Engel

Overview

- Data-flow analyses
 - Forward analyses: Reaching definitions
 - Uninitialized variables analysis
 - Copy propagation

Reaching definitions analysis

- A **definition** of a variable x is a statement which **assigns a value** to x
- A unique **label** (representing the def) is associated with each assignment
 - different occurrences of the same assignment become different definitions
- A definition d **reaches** a point p if there is a path from the point immediately following d to p such that d is not “killed” along that path
- A definition of a variable is **killed** between two points when there is another definition of that variable along the path
 - $r1 = r2 + r3$ kills previous definitions of $r1$

```
d1: y = 3
d2: x = y
```

d1 is a reaching definition for d2

```
d1 : y := 3
d2 : y := 4
d3 : x := y
```

d1 is no longer a reaching definition for d3 because d2 kills its reach

Reaching definitions vs. liveness

- Reaching definitions is different from uses of variables or computation of expressions
 - labels are not associated with them and **hence** lexically same computations are not treated as different entities for analysis
- Liveness
 - analyzes **variables** (e.g., virtual registers)
 - doesn't care about specific users
- Reaching defs
 - analyzes **operations**, each def is different
- **Forward dataflow analysis** as propagation occurs from defs downwards
 - liveness was backward analysis

Data flow equations

- A definition $d_i \in \mathbf{Defs}$ of a variable $x \in \mathbf{Var}$ reaches a program point u if d_i occurs on some path from Start to u and is not followed by any other definition of x on this path
- The data flow equations to define the required analysis are:

$$In_n = \begin{cases} \mathbf{BI} & \text{if } n \text{ is Start block} \\ \bigcup_{p \in \text{pred}(n)} Out_p & \text{otherwise} \end{cases}$$

$$Out_n = (In_n - Kill_n) \cup Gen_n$$

where In_n , Out_n , Gen_n , $Kill_n$, and BI are sets of definitions

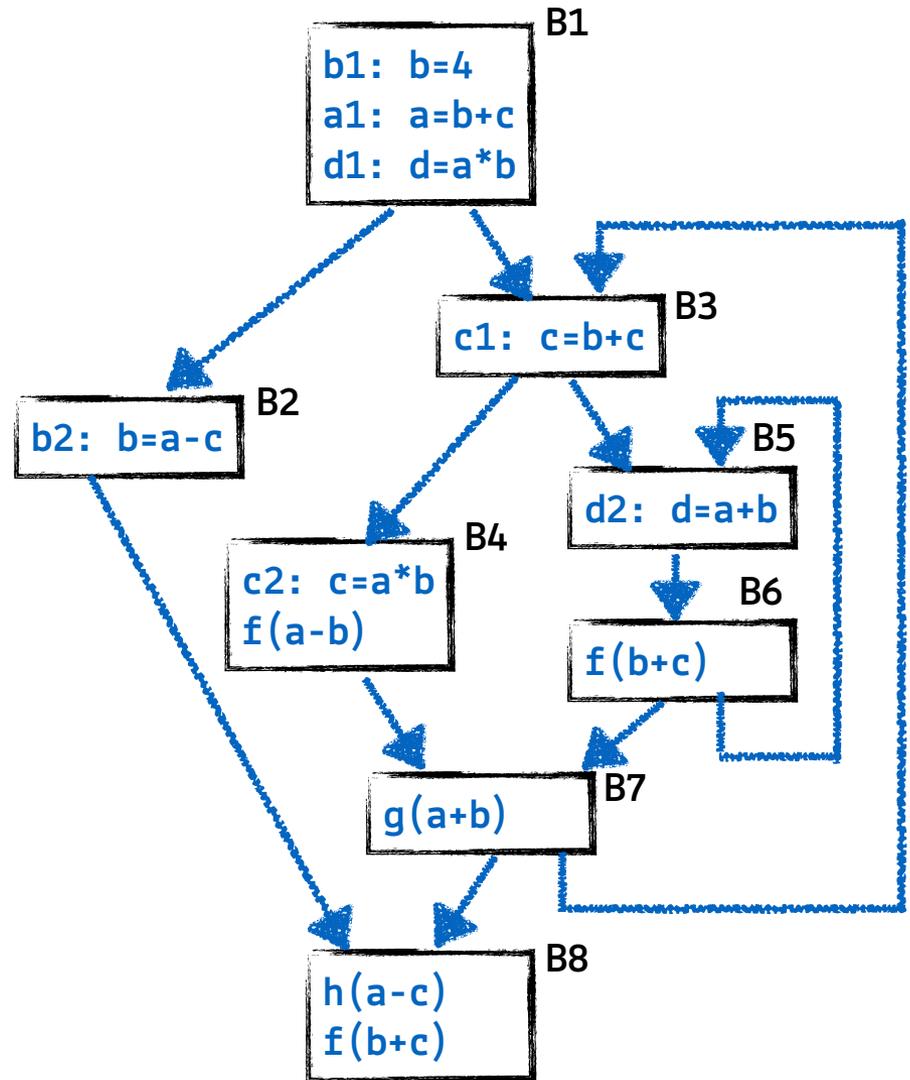
- Note the use of \cup to capture the “*any path*” nature of data flow
 - This is similar to liveness analysis except that now the data flow is forward rather than backward

Assumptions for reaching def. analys.

- For every local variable x , it is assumed that a fictitious definition $x = \text{undef}$ reaches **Entry(Start)**
 - This is required for the optimization of copy propagation (\rightarrow discussed later)
- If definition $x = \text{undef}$ reaches a use of x , it suggests a potential use before definition
- Whether this happens at run time depends on the actual results of conditions along the path taken to reach the program point.
- Gen_n contains downwards exposed definitions in n whereas Kill_n contains all definitions of all variables modified in n
 - Thus $\text{Gen}_n \subseteq \text{Kill}_n$ for reaching definitions analysis

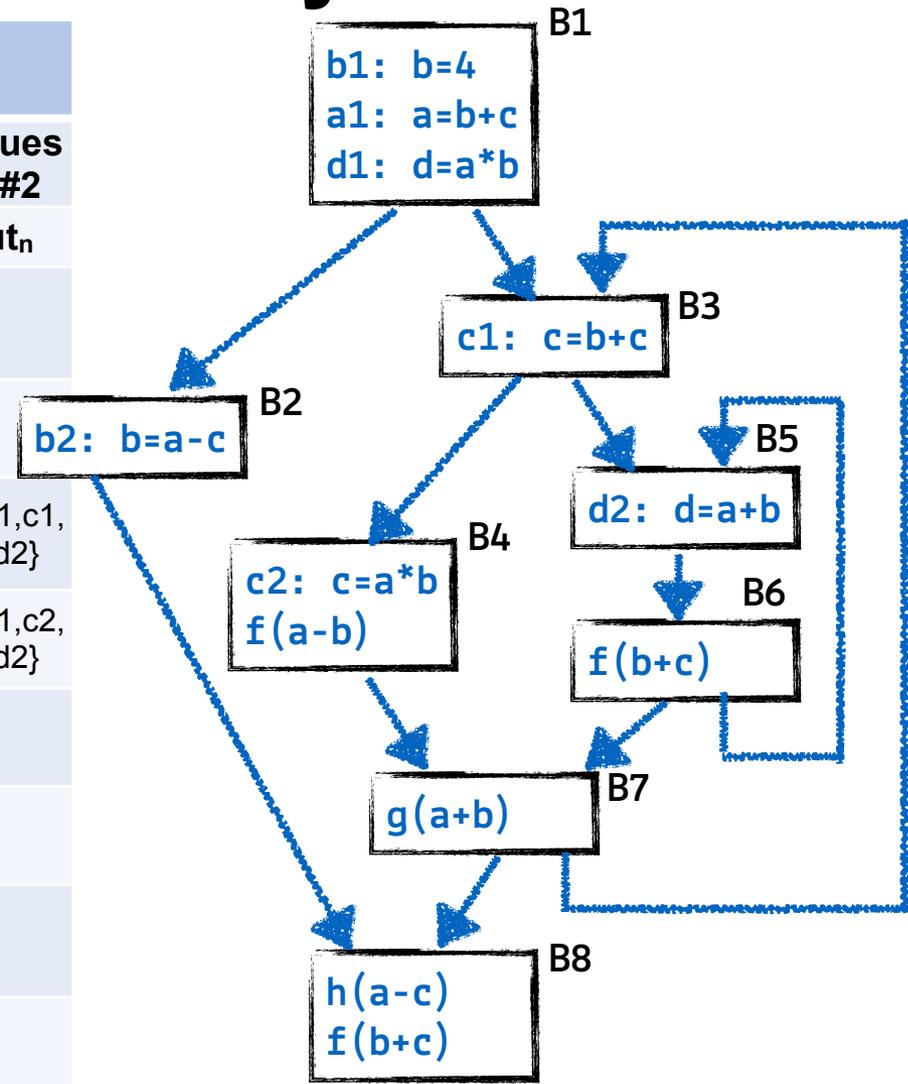
Example

- Labels of assignments consist of variable names and an instance number
 - used to represent the definitions in the programs
- Definitions **a0**, **b0**, **c0**, and **d0** represent the special definitions **a=undef**, **b=undef**, **c=undef**, and **d=undef** respectively
- Since the confluence operation is \cup , the initial value at each program point is \emptyset



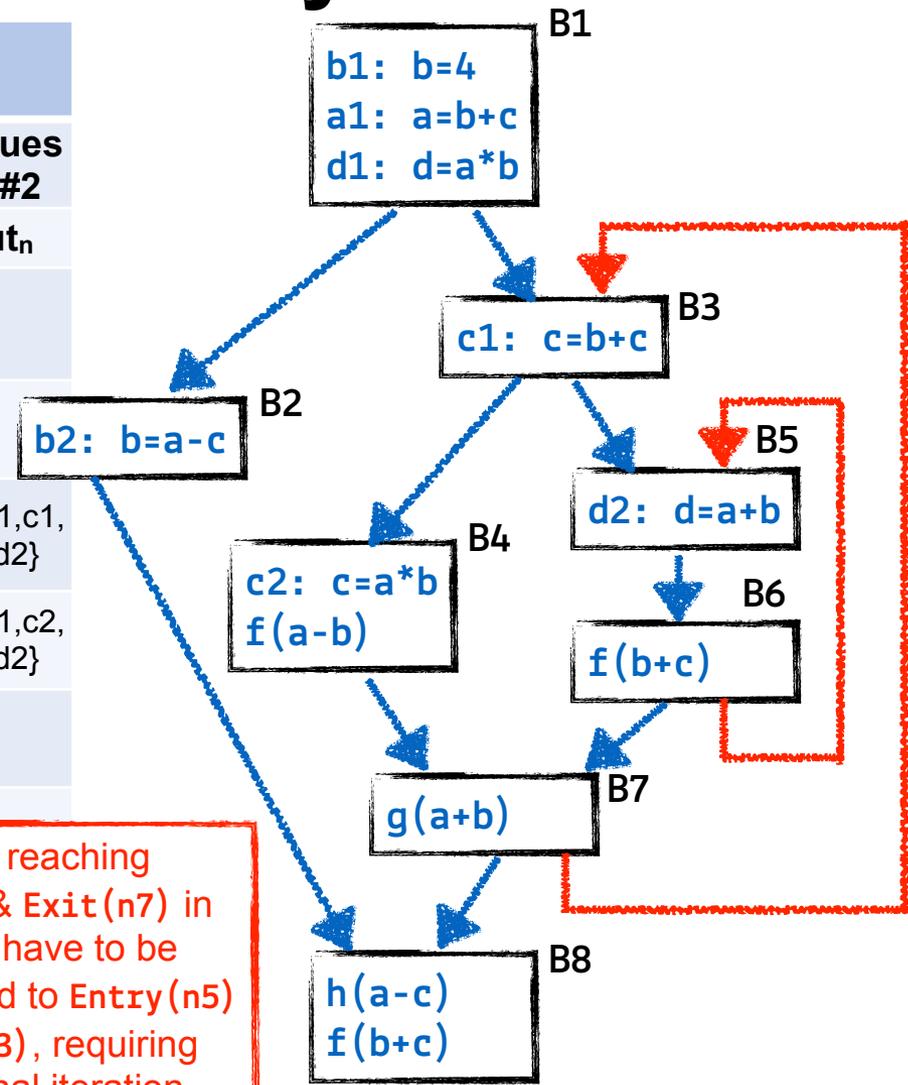
Reaching definitions analysis results

Block	Local information		Global information			
	Gen _n	Kill _n	Iteration #1		Changed values in iteration #2	
			In _n	Out _n	In _n	Out _n
B1	{a1,b1,d1}	{a0,a1,b0,b1,b2,d0,d1,d2}	{a0,b0,c0,d0}	{a1,b1,c0,d1}		
B2	{b2}	{b0,b1,b2}	{a1,b1,c0,d1}	{a1,b2,c0,d1}		
B3	{c1}	{c0,c1,c2}	{a1,b1,c0,d1}	{a1,b1,c1,d1}	{a1,b1,c0,c1,c2,d1,d2}	{a1,b1,c1,d1,d2}
B4	{c2}	{c0,c1,c2}	{a1,b1,c1,d1}	{a1,b1,c2,d2}	{a1,b1,c1,d1,d2}	{a1,b1,c2,d1,d2}
B5	{d2}	{d0,d1,d2}	{a1,b1,c1,d1}	{a1,b1,c1,d2}	{a1,b1,c1,d1,d2}	
B6	∅	∅	{a1,b1,c1,d1}	{a1,b1,c1,d2}		
B7	∅	∅	{a1,b1,c1,c2,d1,c2}	{a1,b1,c1,c2,d1,c2}		
B8	∅	∅	{a1,b1,c1,c2,d1,d2}	{a1,b1,b2,c0,b2,c0,c1,c2,d1,d2}		



Reaching definitions analysis results

Block	Local information		Global information			
	Gen _n	Kill _n	Iteration #1		Changed values in iteration #2	
			In _n	Out _n	In _n	Out _n
B1	{a1,b1,d1}	{a0,a1,b0,b1,b2,d0,d1,d2}	{a0,b0,c0,d0}	{a1,b1,c0,d1}		
B2	{b2}	{b0,b1,b2}	{a1,b1,c0,d1}	{a1,b2,c0,d1}		
B3	{c1}	{c0,c1,c2}	{a1,b1,c0,d1}	{a1,b1,c1,d1}	{a1,b1,c0,c1,c2,d1,d2}	{a1,b1,c1,d1,d2}
B4	{c2}	{c0,c1,c2}	{a1,b1,c1,d1}	{a1,b1,c2,d2}	{a1,b1,c1,d1,d2}	{a1,b1,c2,d1,d2}
B5	{d2}	{d0,d1,d2}	{a1,b1,c1,d1}	{a1,b1,c1,d2}	{a1,b1,c1,d1,d2}	
B6	∅	∅	{a1,b1,c1,d1}	{a1,b1,c1,d2}		
B7	∅	∅	{a1,b1,c1,c2,d1,c2}	{a1,b1,c1,c2,d1,c2}		
B8	∅	∅	{a1,b1,c1,c2,d1,d2}	{a1,b1,b2,c0,c1,c2,d1,d2}		



definitions reaching Exit(n6) & Exit(n7) in iteration 1 have to be propagated to Entry(n5) & Entry(n3), requiring an additional iteration

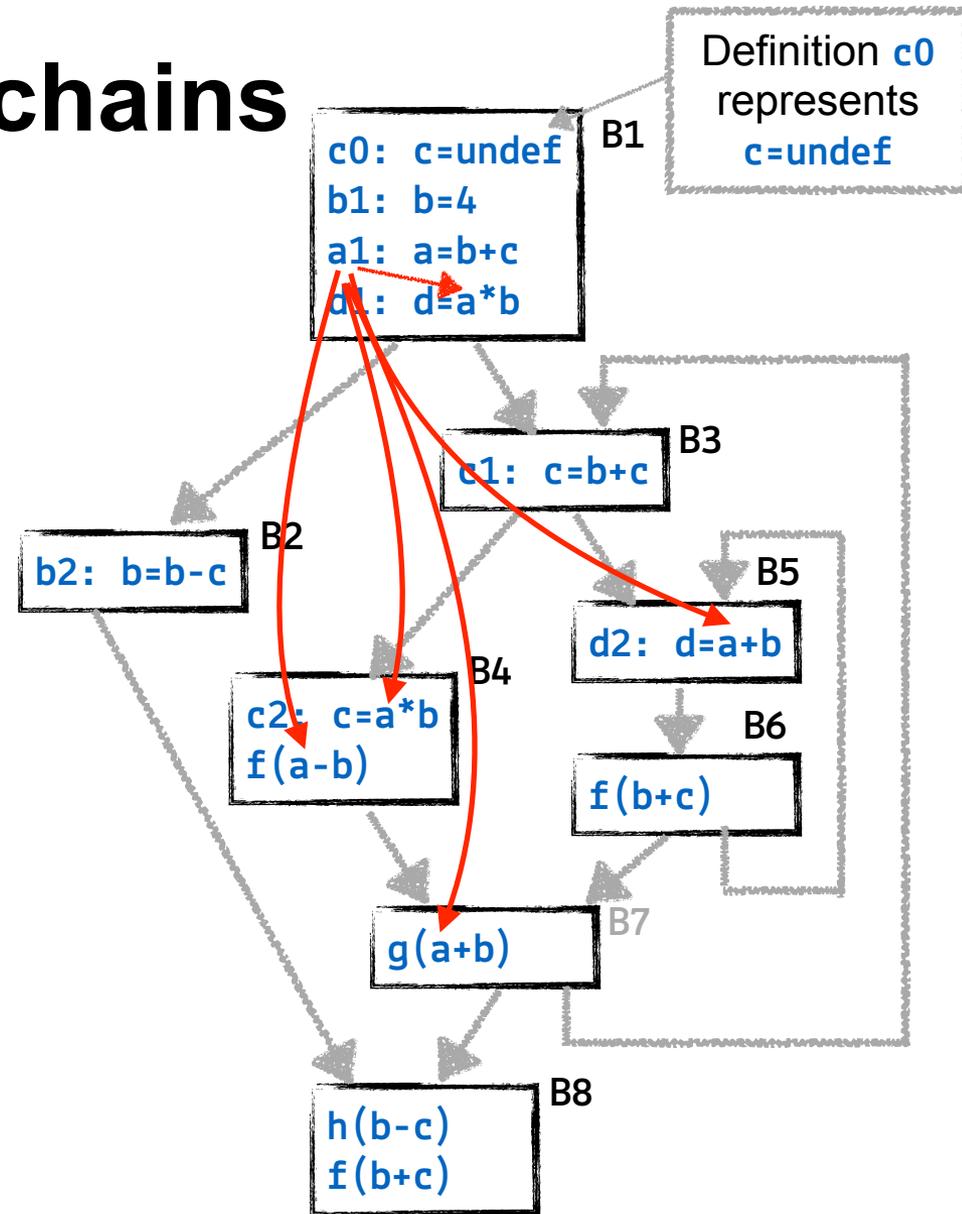
def-use & use-def chains

- Reaching definitions analysis is used for constructing use-def and def-use chains which connect definitions to their uses
 - These chains facilitate several optimizing transformations

Example:

def-use chain for variable **a**

- Chains always start at a *label*

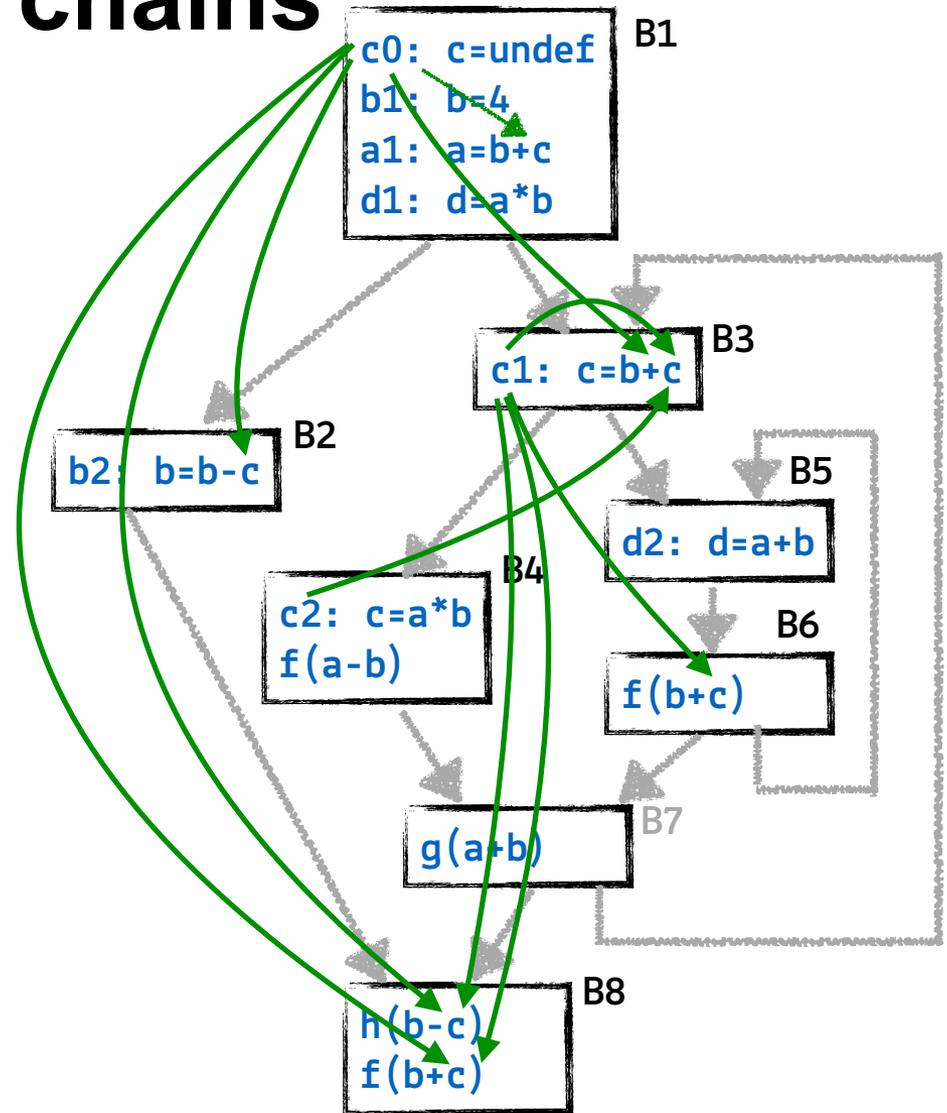


def-use & use-def chains

Example:

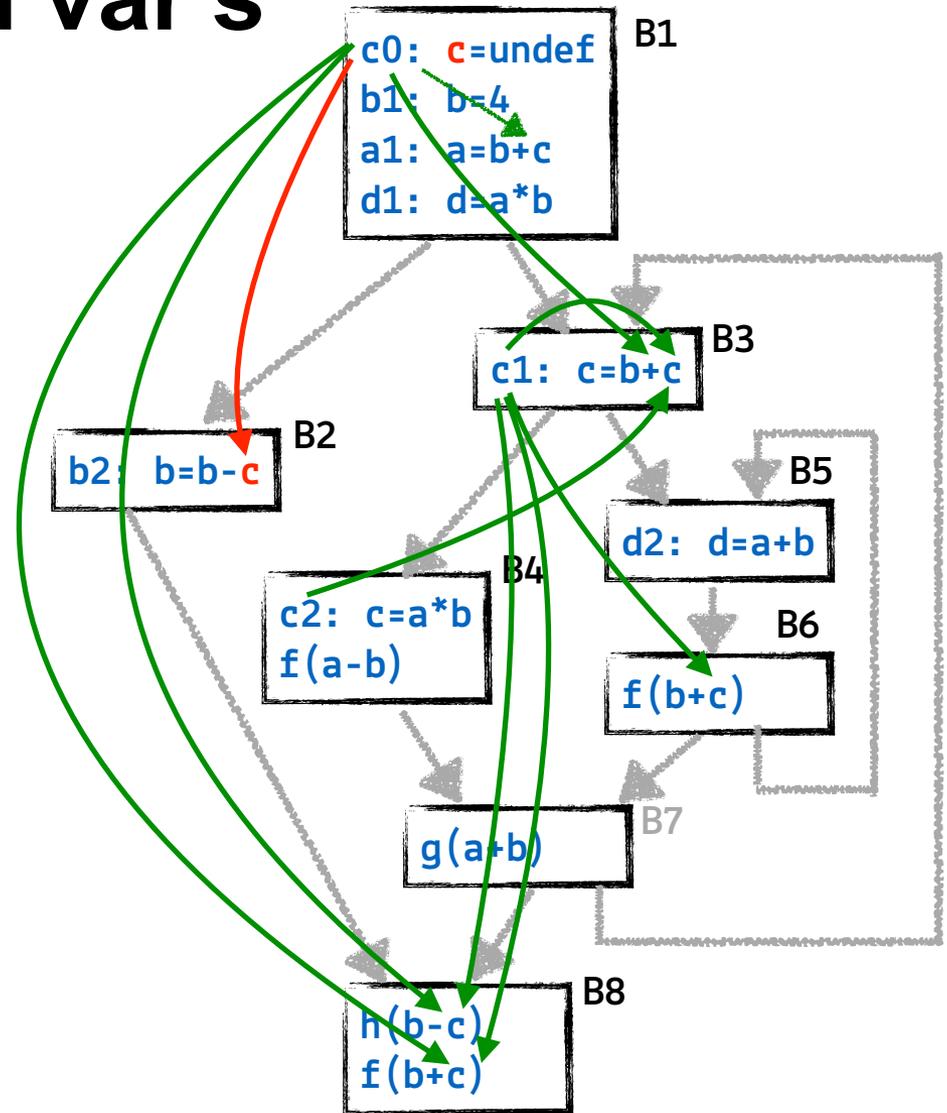
def-use chain for variable **c**

- Definition **c0** reaches some uses of **c**
- This suggests a potential use before any assigning meaningful value
- This, in turn, makes variable **b** *potentially undefined*



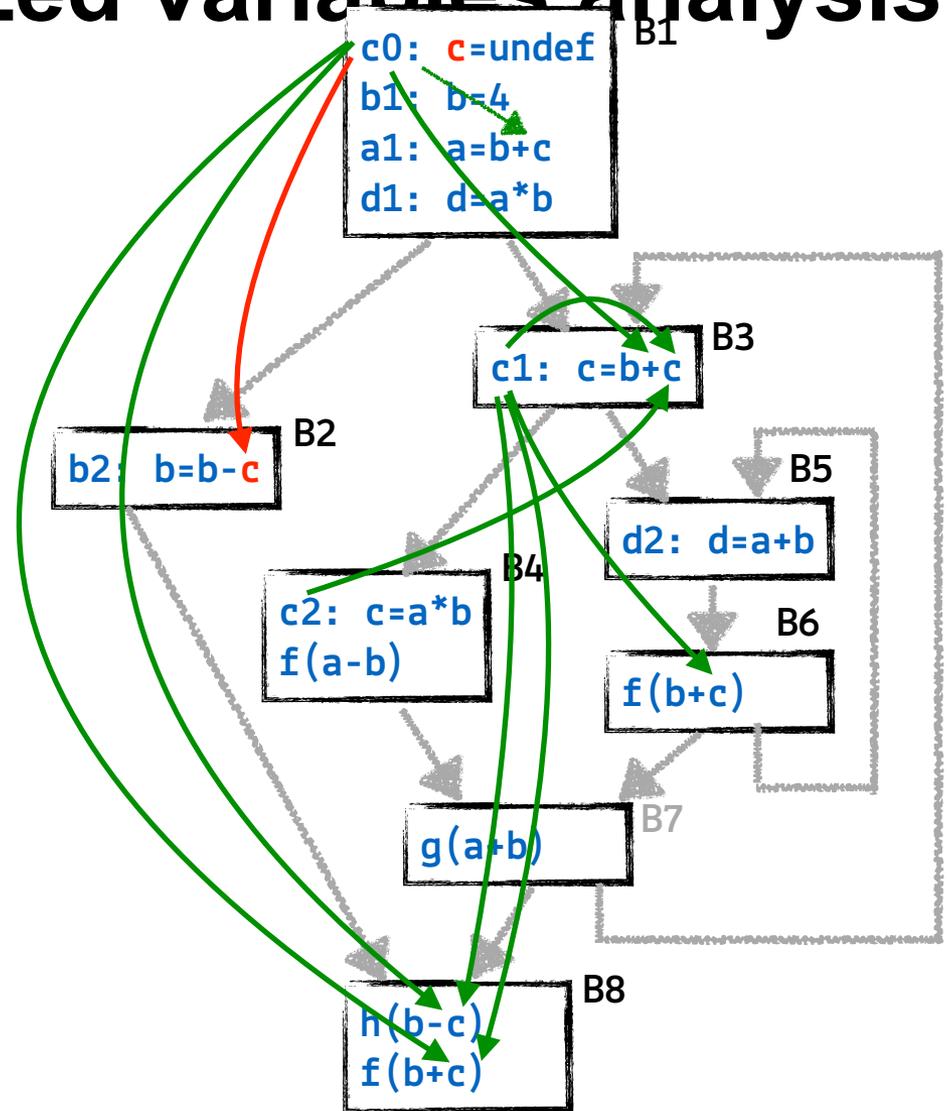
Finding undefined var's

- Definition `c0` reaches some uses of `c`
- This, in turn, makes variable `b` **potentially undefined**
- **Transitive effects** of undefined variables are captured by **possibly uninitialized variables analysis**
- Possibly uninitialized variables analysis is non-separable – whether a variable is possibly undefined may depend on whether other variables are possibly undefined.



Possibly uninitialized variables analysis

- For definition x_i of variable x , reaching definitions analysis discovers a set of **definition reaching paths**:
- a sequence of blocks (b_1, b_2, \dots, b_k) which is a prefix of some potential execution path starting at b_1 such that:
 - b_1 contains the definition x_i
 - b_k is either **End** or contains a definition of x
 - no other block in the path contains a definition of x
- **Example:** some definition reaching paths for variable c are:
 (B_4, B_7, B_3) , $(B_3, B_5, B_6, B_7, B_3)$ and $(B_3, B_5, B_6, B_5, B_6, B_7, B_8)$



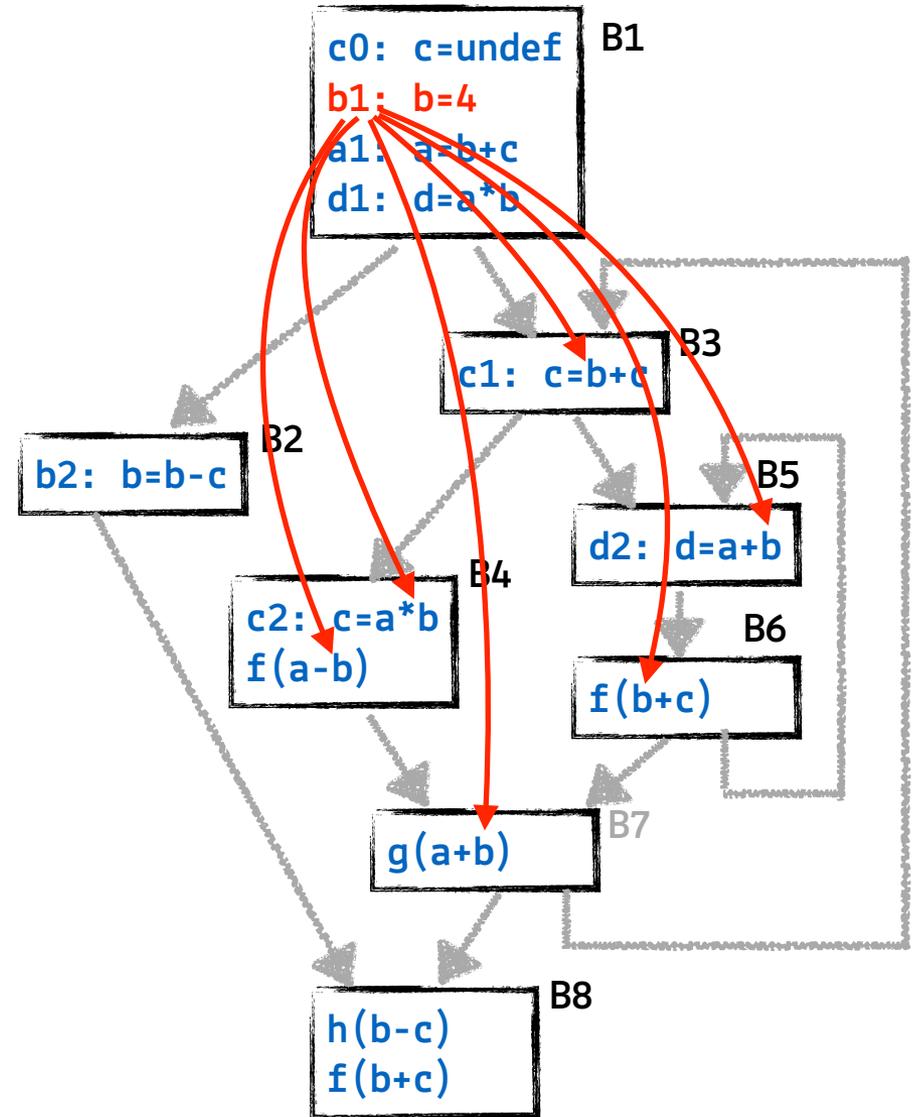
Reaching def. for copy propagation

- Another application of reaching definitions analysis is in performing **copy propagation**
- A definition of the form $x=y$ is called a **copy** because it merely copies the value of y to x
- When such a definition reaches a use of x , and no other definition of x reaches that use then the use of x can be replaced by y

Copy propagation

Example:

- Copy $b=4$ in block **B1** is the only definition which reaches the uses of b in blocks **B3**, **B4**, **B5**, **B6** and **B7**
- Thus all these uses can be replaced by the constant 4



Copy propagation

```
c0: c=undef
b1: b=4
a1: a=b+c
d1: d=a*b
```

- In the above example, the right hand side (RHS) value is constant
- With variables on the RHS, e.g. $x=y$, replacing the uses of x by y requires an additional check to ensure that the value of y has not been modified along the path from copy to use

A variant of our reaching definitions analysis can accomplish this:

- We restrict the defs to **copies**, a def $x=y$ is contained in:
 - Gen_n if it is downwards exposed in n , i.e. not being followed by a definition of x or y , and in
 - $Kill_b$ if n contains a definition of x or y
- We can now perform reaching definitions analysis
- If one def reaches a use, we can perform **copy propagation**

Use of copy propagation

- This copy propagation optimization does not improve the program on its own
- But it has the potential of creating **dead code**:
 - When copy propagation is performed using $x = y$, it is possible that all uses of x are replaced by y thus making x **dead** after the assignment
 - Thus this assignment can be safely deleted

References

- [1] Allen, Frances E. and Cocke, John. A catalogue of optimizing transformations. RC 3548, IBM T. J. Watson Research Center, Yorktown Heights, N.Y., September 1971