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Compiler Construction

Lecture 19–2: Live variable analysis

Week of 2020-03-16
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Overview

- Data-flow analyses
  - Backward analyses: Live variable analysis

Live variable analysis

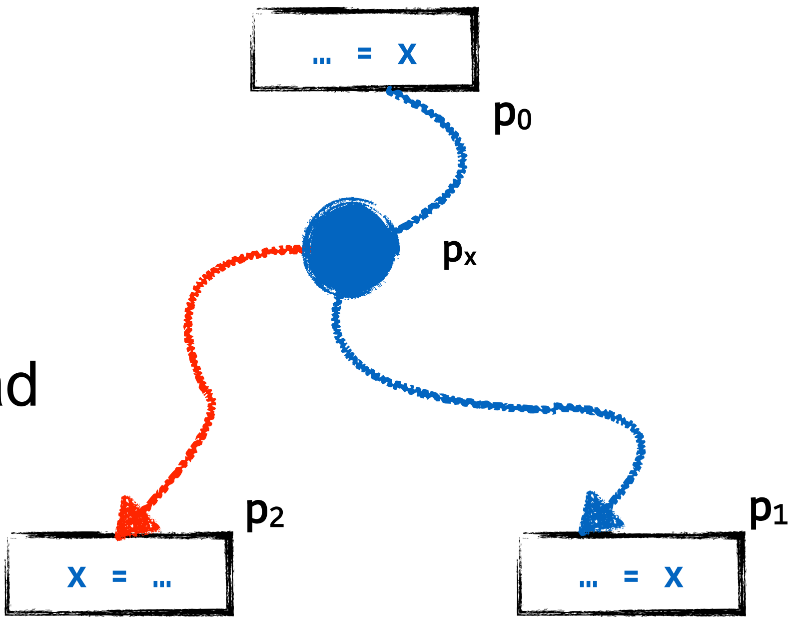
What is Live Variable Analysis?

- For each variable x we determine:
  Where is the last program point p at which a specific value of x is used?
- In other words:
  For x and a program point p determine if the value of x at p can still be used along some path starting at p
  - If so, x is **live** at p
  - If not, x is **dead** at p
- Live variable analysis must take control flow into account
  ⇒ we need to solve a data flow problem

Live variable analysis: example

At point p_0 variable **x** is live:

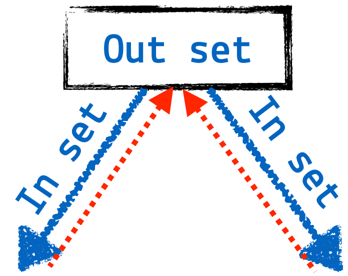
- There is a path to p_1 where the value at p_0 is used
- Beyond p_x towards p_2 the value of **x** is no longer needed and is dead



For each variable and for each program point, we have to observe:

- Where is the last program point beyond which the value is not used?
- **Trace back** from uses to definitions and observe the first definition (backwards) that reaches that use
- That definition **kills** all uses backwards of it

Gen and kill, in and out sets



- A variable is **live** at a point p if its value is used along at least one path
  - A use of x prior to any definition in a basic block means x must be alive
  - A definition of x in a block B prior to any subsequent use means previous uses must be dead
- Accordingly, we obtain:
  - ***Gen**:* set of variables **used** in B
    - the upward exposed reads of variables in block B
  - ***Kill**:* set of variables **defined** in B

$$\boldsymbol{Out_b} = \bigcup_{s \in succ(b)} \boldsymbol{In_s} \qquad\qquad \boldsymbol{In_b = Use_b \cup (Out_b - Def_b)}$$

Implementing live variables analysis

- Initialize $\boldsymbol{In_b}$ to the empty set
- Compute **Gen/Use** and **Kill/Def** for each basic block
  - Tracing backwards from the end of the block to the beginning of the block
  - Initialize **last instruction's** $\boldsymbol{Out_i}$ to the empty set
  - Apply $\boldsymbol{In_i = Use_i \cup (Out_i - def_i)}$
- Iteratively apply relations to basic block until convergence
  - $$\boldsymbol{Out_b} = \bigcup_{s \in succ(b)} \boldsymbol{In_s}$$
  - $\boldsymbol{In_b = Use_b \cup (Out_b - def_b)}$
- With $\boldsymbol{Out_b}$, use relations at instruction level to determine the live variables after each instruction

Compute use and def for a basic block

`t = a + 1`	Use = {a} Def = {t}
`b = t`	Use = {t} Def = {b}
`if (a=b) goto L2`	Use = {a,b} Def = {}

$$In = Use \cup (Out - def)$$

$$Out = \{\}$$

$$In_i = Use_i \cup (Out_i - def_i)$$

Compute use and def for a basic block

```
t = a + 1
```
Use = {a}
Def = {t}

```
b = t
```
Use = {t}
Def = {b}

```
if (a=b) goto L2
```
Use = {a,b}
Def = {}

In **=** {a,b} ∪ ({} - {}) = {a,b}

Out **=** {}

$$In_i = Use_i \cup (Out_i - def_i)$$

Compute use and def for a basic block

`t = a + 1`	Use = {a} Def = {t}
`b = t`	Use = {t} Def = {b}
`if (a=b) goto L2`	Use = {a,b} Def = {}

In* = *Use* $\cup$ *(Out - def)

***Out* = {a,b}**

***Out* = {}**

$$In_i = Use_i \cup (Out_i - def_i)$$

Compute use and def for a basic block

```
t = a + 1
```
Use = {a}
Def = {t}

```
b = t
```
Use = {t}
Def = {b}

```
if (a=b) goto L2
```
Use = {a,b}
Def = {}

In **=** {t} ∪ ({a,b} - {b}) = {a,t}

Out **=** {a,b}

Out **=** {}

$$In_i = Use_i \cup (Out_i - def_i)$$

Compute use and def for a basic block

```
t = a + 1
```
Use = {a}
Def = {t}

Out = {a,t}

```
b = t
```
Use = {t}
Def = {b}

Out = {a,b}

```
if (a=b) goto L2
```
Use = {a,b}
Def = { }

Out = {}

$$In = Use \cup (Out - def)$$

$$In_i = Use_i \cup (Out_i - def_i)$$

Compute use and def for a basic block

```
t = a + 1
```

Use = {a}
Def = {t}

In = {a} ∪ ({a,t} - {t}) = {a}

Out = {a,t}

```
b = t
```

Use = {t}
Def = {b}

Out = {a,b}

```
if (a=b) goto L2
```

Use = {a,b}
Def = { }

Out = {}

$$In_i = Use_i \cup (Out_i - def_i)$$

Compute use and def for a basic block

		In **=** {a} ∪ ({a,t} - {t}) = **{a}**
`t = a + 1`	Use = {a} Def = {t}	
		Out **=** {a,t}
`b = t`	Use = {t} Def = {b}	
		Out **=** {a,b}
`if (a=b) goto L₂`	Use = {a,b} Def = {}	
		Out **=** {}

$$\boldsymbol{InUse_i = Use_i \cup (OutUse_i - defUse_i)}$$

$$Use_b = \{a\}$$

Compute use and def for a basic block

```
t = a + 1
```

Use = {a}
Def = {t}

```
b = t
```

Use = {t}
Def = {b}

```
if (a=b) goto L2
```

Use = {a,b}
Def = {}

***OutDef* = {}**

$$InDef_i = Def_i \cup (OutDef_j)$$

Compute use and def for a basic block

```
t = a + 1
```

Use = {a}
Def = {t}

```
b = t
```

Use = {t}
Def = {b}

```
if (a=b) goto L2
```

Use = {a,b}
Def = {}

$InDef = Def \cup (OutDef) = \{\}$

$OutDef = \{\}$

$$InDef_i = Def_i \cup (OutDef_i)$$

Compute use and def for a basic block

```
t = a + 1
```
Use = {a}
Def = {t}

InDef = *Def* ∪ *(OutDef)* = {b}

```
b = t
```
Use = {t}
Def = {b}

OutDef = {}

```
if (a=b) goto L2
```
Use = {a,b}
Def = { }

OutDef = {}

$$InDef_i = Def_i \cup (OutDef_j)$$

Compute use and def for a basic block

```
t = a + 1
```

Use = {a}
Def = {t}

```
b = t
```

Use = {t}
Def = {b}

```
if (a=b) goto L2
```

Use = {a,b}
Def = {}

InDef = Def $\cup$ ***(OutDef)***

= {t} $\cup$ {b}

OutDef = {b}

OutDef = {}

OutDef = {}

$$InDef_i = Def_i \cup (OutDef_j)$$

Compute use and def for a basic block

InDef = {t, b}

```
t = a + 1
```

Use = {a}
Def = {t}

OutDef = {b}

```
b = t
```

Use = {t}
Def = {b}

OutDef = {}

```
if (a=b) goto L2
```

Use = {a,b}
Def = {}

OutDef = {}

$InDef_i = Def_i \cup (OutDef_j)$

Def_b = {t,b}

Liveness semantics

Assuming that variable **x** is live at the exit of basic block **n**, there are four possibilities with four distinct semantics:

Case	Local information		Effect on liveness
1	x $\notin$ ***Gen***$_n$	x $\notin$ ***Kill***$_n$	Liveness of **x** is unaffected in block **n**
2	x $\in$ ***Gen***$_n$	x $\notin$ ***Kill***$_n$	Liveness of **x** is generated in block **n**
3	x $\notin$ ***Gen***$_n$	x $\in$ ***Kill***$_n$	Liveness of **x** is killed in block **n**
4	x $\in$ ***Gen***$_n$	x $\in$ ***Kill***$_n$	Liveness of **x** is unaffected in block **n** in spite of **x** being modified in **n**

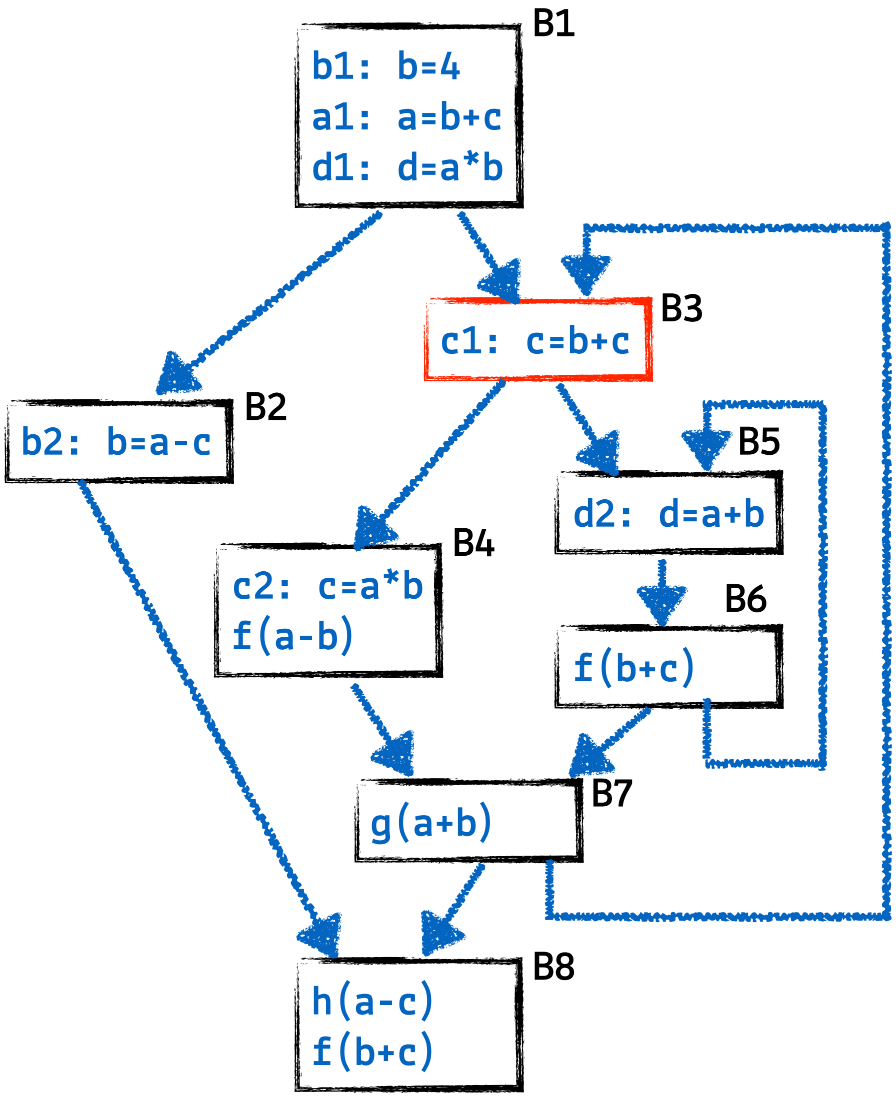
- Variable **x** is live at `Entry(n)` in cases 1, 2, and 4 but the reason for its liveness is different in each case
- Case 4 captures the fact that the liveness of **x** is killed in **n** but is regenerated within **n**

```
a = b + c
c = 42
```

Example

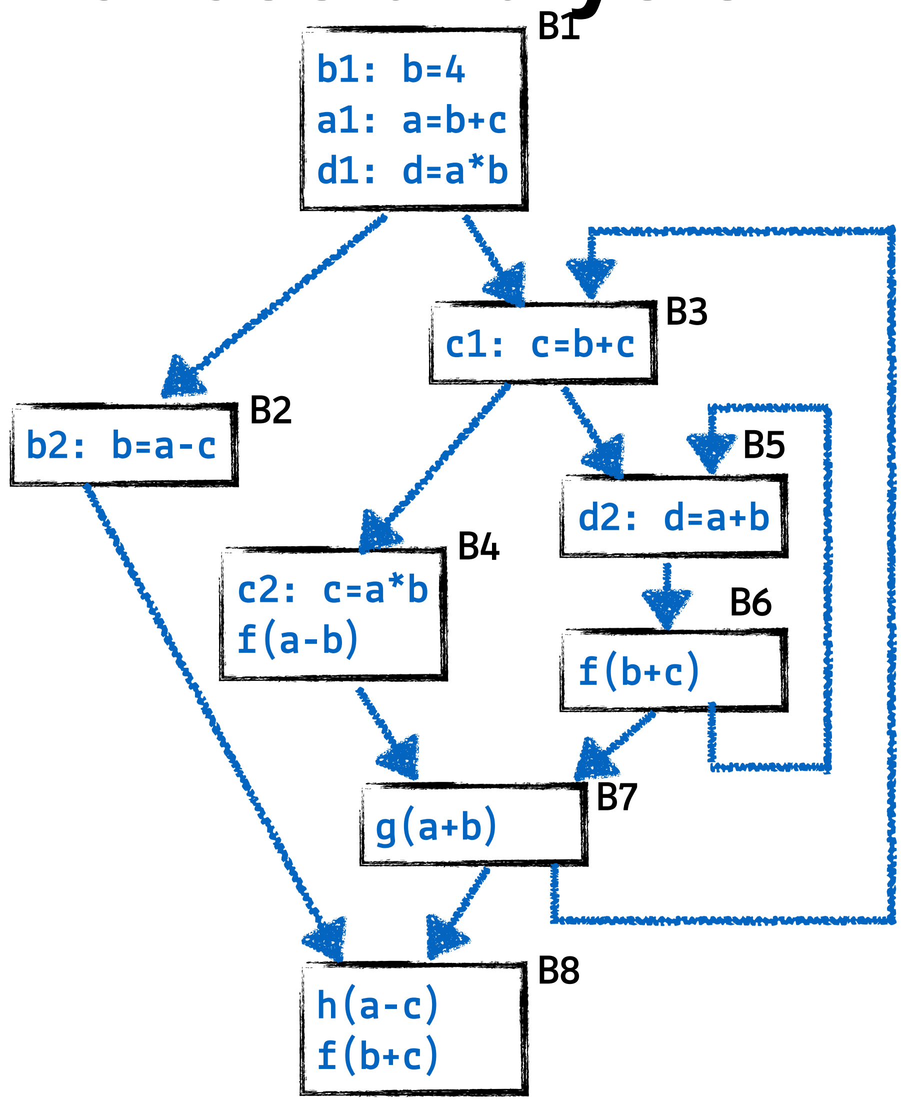
Var = {a,b,c,d}
Defs = {a1,b1,b2,c1,c2,d1,d2}
Expr = {a∗b,a+b,a-b,a-c,b+c}

- Variable c is contained in both ***Gen₃*** and ***Kill₃***



Example: trace of liveness analysis

Block	Local information		Global information			
			Iteration #1		Iteration #2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
B8	{a,b,c}	ø	ø	{a,b,c}	ø	{a,b,c}
B7	{a,b}	ø	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B6	{b,c}	ø	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B5	{a,b}	{d}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B4	{a,b}	{c}	{a,b,c}	{a,b}	{a,b,c}	{a,b}
B3	{b,c}	{c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B2	{a,c}	{b}	{a,b,c}	{a,c}	{a,b,c}	{a,c}
B1	{c}	{a,b,d}	{a,b,c}	{c}	{a,b,c}	{c}



Example: trace of liveness analysis

Block	Local information		Global information			
			Iteration #1		Iteration #2	
	Gen_n	$Kill_n$	Out_n	In_n	Out_n	In_n
B8	{a,b,c}	ø	ø	{a,b,c}	ø	{a,b,c}
B7	{a,b}	ø	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B6	{b,c}	ø	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B5	{a,b}	{d}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B4	{a,b}	{c}	{a,b,c}	{a,b}	{a,b,c}	{a,b}
B3	{b,c}	{c}	{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}
B2	{a,c}	{b}	{a,b,c}	{a,c}	{a,b,c}	{a,c}
B1	{c}	{a,b,d}	{a,b,c}	{c}	{a,b,c}	{c}

The data flow values computed in iteration #2 are identical to the values computed in iteration #1 ⇒ ***convergence***

The result would be ***different*** if we had used the ***universal set*** (here: `{a,b,c,d}`) as initialization. Then, `d` would have been live at `Exit(B7)` whereas `d` is not used anywhere in the program

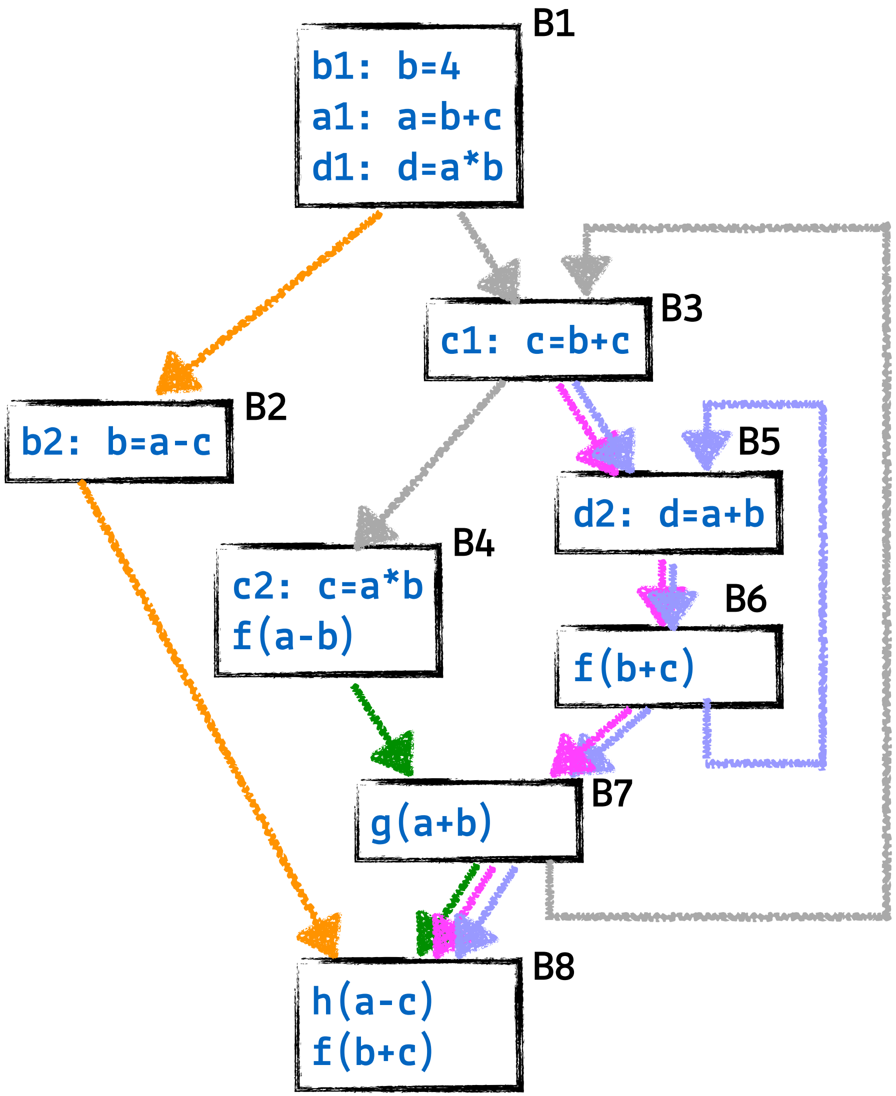
Liveness paths

- For a given variable **x**, liveness analysis discovers a set of **liveness paths**
- Each liveness path is a sequence of blocks (B_1, B_2,..., B_k) which is a prefix of some potential execution path starting at B_1 such that:
  - B_k contains an upwards exposed use of **x**, and
  - **x** is either **Start** or contains an assignment to **x**, and
  - no other block on the path contains an assignment to **x**

Liveness paths

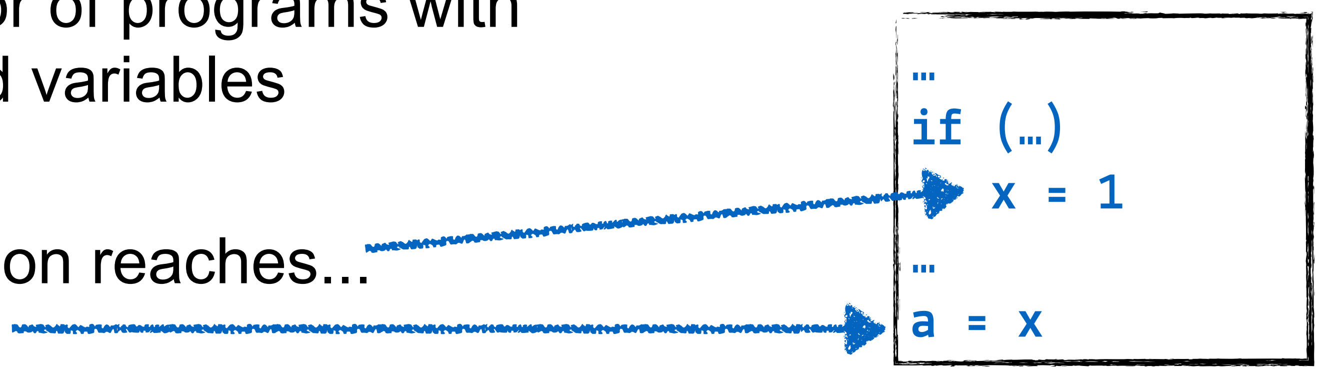
- Some liveness paths for variable `c` in our example program are:

  (B4,B7,B8),
  (B3,B5,B6,B7,B8),
  (B3,B5,B6,B5,B6,B7,B8),
  and (B1,B2,B8)



Applications of liveness analysis

- **Finding uninitialized variables**:
  - Languages like C typically do not define the behavior of programs with uninitialized variables
  - This definition reaches...
    this use…
    but the def might not get executed!



- Common source of security problems [2]

Applications of liveness

- **Dead code elimination**:
  - If **x** is not live at a exit of an assignment of **x**, then this assignment can be safely deleted
- Discover useless store operations
  - At an operation that stores **v** to memory, if **v** is not live then the store is useless
- In the example, the assignments `global=1` and `global=3` assign to dead variables
- `i` is not live at the end of `f`, so the assignment can be eliminated

```
int global;
void f ()
{
  int i;
  // dead store:
  i = 1;
  // dead store:
  global = 1;
  global = 2;
  return;
  // unreachable:
  global = 3;
}
```

↓

```
int global;
void f ()
{
  global = 2;
  return;
}
```

Applications of liveness analysis

- **Register allocation**:
  - If a variable **x** is live at a program point, the current value of **x** is likely to be used along some execution path and hence **x** is a potential candidate for being allocated a register
  - On the other hand, if **x** is not live, the register allocated to **x** can be allocated to some other variable without the need of storing the value of **x** in memory
  - More details on register allocation later

Dead variables analysis

- A variable is ***dead*** (i.e., not live) if it is dead ***along all paths***
- We can perform dead variables analysis instead of live variables analysis
- The interpretation of ***Inn*** and $\boldsymbol{Out_n}$ changes
  - If a variable is contained in $\boldsymbol{In_n}$ or $\boldsymbol{Out_n}$, it is dead instead of being live

References

[1] J. C. Beatty (1975).
An algorithm for tracing live variables based on a straightened program graph,
International Journal of Computer Mathematics, 5:1-4, 97-108,
DOI: 10.1080/00207167508803104
[2] http://cwe.mitre.org/data/definitions/457.html